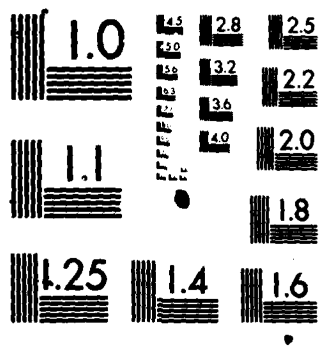
AD-A114 625 WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER F/G 12/1
DISTRIBUTED BILINEAR SYSTEMS: POSITIVE AND NEGATIVE RESULTS ON --ETC(U)
DEC 81 M SLEMROD DAAG29-80-C-0041
UNCLASSIFIED MRC-TSR-2310 NL

1 of 1

END
DATE
FILMED
6-82
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

2

ADA114625

MRC Technical Summary Report #2310

DISTRIBUTED BILINEAR SYSTEMS: POSITIVE AND NEGATIVE RESULTS ON CONTROLLABILITY

M. Slemrod

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53706

December 1981

(Received November 23, 1981)

Approved for public release
Distribution unlimited

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

and

Air Force Office
of Scientific Research
Washington, DC 20332

UNIVERSITY OF WISCONSIN - MADISON
MATHEMATICS RESEARCH CENTER

DISTRIBUTED BILINEAR SYSTEMS: POSITIVE AND NEGATIVE RESULTS ON CONTROLLABILITY

M. Slemrod*

Technical Summary Report #2310

December 1981

ABSTRACT

This paper outlines results presented in [2] on the problem of controlling bilinear distributed parameter systems. Specifically we give results showing that one cannot control a bilinear distributed parameter system to a full open neighborhood of an infinite dimensional state space. Nevertheless we do present a result which allows identification of an accessible set of states for a class of "hyperbolic" control systems.

AMS (MOS) Subject Classification: 93B05, 93C10, 93C20, 93C[illegible]

Key Words: Distributed control system, bilinear system, hyperbolic partial differential equations, rod equation.

Work Unit Number 1 - Applied Analysis

---

* Dept. of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, NY 12181 and the Mathematics Research Center, University of Wisconsin, Madison, WI 53706.

Research sponsored in part by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Contract/Grant No. AFOSR-81-0172. The United States Government is authorized to produce and distribute reprints for Government purposes not withstanding any copyright notation hereon. Support facilities provided by the United States Army under Contract No. DAAG29-80-C-0041.

SIGNIFICANCE AND EXPLANATION

One way to view control of linear partial differential equations where the controls enter as time varying coefficients is as a bilinear distributed system. For example the problem of controlling a rod via the axial load falls in this class. This paper examines the possibility of using such controls to steer the system from one location to another. It shows that this problem is generally ill-posed but may be solved in certain exceptional circumstances.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

CONTENTS

1. Distributed bilinear control systems

2. The control problem

3. Identification of the accessible set for abstract "hyperbolic" bilinear control systems

4. References

DISTRIBUTED BILINEAR SYSTEMS: POSITIVE AND NEGATIVE RESULTS IN CONTROLLABILITY

M. Slemrod*

1. Distributed bilinear control systems

By a distributed bilinear control system we mean a system of the form

$$\dot{\omega}(t) = \mathcal{A}\omega(t) + p(t)\mathcal{B}\omega(t) \quad , \tag{1.1}$$

$$\omega(0) = \omega_0 \in X \quad , \tag{1.2}$$

where $\mathcal{A}$ generates a C^0 semigroup of bounded linear operators on a (possible complex) Banach space X, $\mathcal{B} : X \to X$ is a bounded linear operator, and $p \in L^1([0,T]; R)$ is a real valued control defined on a specified interval $[0,T]$.

Of particular interest in applications is the abstract "hyperbolic" bilinear control system given by

$$\ddot{u}(t) + Au(t) + p(t)Bu(t) = 0 \quad , \tag{1.3}$$

$$u(0) = u_0 \in D(A^{1/2}),\ \dot{u}(0) = u_1 \in H \quad , \tag{1.4}$$

where A is a positive definite self-adjoint operator with dense domain $D(A)$ in a real Hilbert space H, B is a bounded linear operator from $D(A^{1/2})$ to H, and p (the control) is again in $L^1([0,T]; R)$. We suppose A^{-1} is compact and A has simple eigenvalues λ_n^2, $n = 1,2,\ldots,$ where $0 < \lambda_1 < \lambda_2 < \cdots$. Then there exists a corresponding sequence $\{\phi_n\}$ of eigenfunctions: $A\phi_n = \lambda_n^2\phi_n$, $\langle\phi_n,\phi_m\rangle_H = \delta_{mn}$ where $\langle\cdot,\cdot\rangle_H$ denotes the inner product in H.

* Dept. of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, NY 12181 and the Mathematics Research Center, University of Wisconsin, Madison, WI 53706.

Research sponsored in part by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Contract/Grant No. AFOSR-81-0172. The United States Government is authorized to produce and distribute reprints for Government purposes not withstanding any copyright notation hereon. Support facilities provided by the United States Army under Contract No. DAAG29-80-C-0041.

While we could rewrite (1.3), (1.4) in first order form (1.1), (1.2) in the usual manner of dynamical systems (see for example [1], [2]) it is preferable for our purposes to introduce a complex structure similar to that used for Hamiltonian systems.

Let $\mathcal{H}$ denote the complexified Hilbert space $H \oplus iH$ with inner product defined by

$$\langle x_1 + iy_1, x_2 + iy_2\rangle = \langle x_1,x_2\rangle_H + \langle y_1,y_2\rangle_H + i[\langle y_1,x_2\rangle_H - \langle x_1,y_2\rangle_H]$$

for $x_1,x_2,y_1,y_2 \in H$. Let $z(t) = A^{1/2}u(t) + i\dot{u}(t)$. Then (1.3), (1.4) becomes

$$\dot{z}(t) = \hat{A}z(t) + p(t)\hat{B}z(t) \quad , \tag{1.5}$$

$$z(0) = A^{1/2}u_0 + iu_1 \in \mathcal{H} \quad , \tag{1.6}$$

where $\hat{A} = -iA^{1/2}$, $\hat{B} = -iBA^{-1/2}\mathrm{Re}$, and thus (1.3), (1.4) has been put in the form (1.1), (1.2) with $X = \mathcal{H}$.

<u>Example.</u> The rod equation with hinged ends.

Consider the system

$$u_{tt} + u_{xxxx} + p(t)u_{xx} = 0, \quad 0 < x < 1 \quad , \tag{1.7}$$

with boundary conditions

$$u = u_{xx} = 0 \quad \text{at} \quad x = 0,1 \quad , \tag{1.8}$$

with initial conditions

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), \quad 0 < x < 1 \quad . \tag{1.9}$$

In the notation of (1.3), (1.4) we have

$$A = \frac{d^4}{dx^4},\ B = \frac{d^2}{dx^2},\ H = L^2(0,1),\ D(A) = \{u \in H^4(0,1);$$

$$u,\ u_{xx} \in H_0^1(0,1)\},\ D(A^{1/2}) = H^2(0,1) \cap H_0^1(0,1),\ \lambda_n = n^2\pi^2,$$

$$\phi_n = \sqrt{2}\sin n\pi x,\ n = 1,2,\ldots \quad .$$

Here $p(t)$ represents the axial load on the rod.

2. The control problem

Consider the system (1.1), (1.2). The controllability problem is

(P) Given $h \in X$, find $p \in L^1([0,T]; R)$ so that the (generalized) solution of (1.1), (1.2) with control denoted by $\omega(t;p,\omega_0)$ satisfies $\omega(T;p,\omega_0) = h$.

We note two important features of (P). First we observe that even though (1.1) is a linear evolution equation in ω for fixed p the map $\omega(T;\cdot,\omega_0)$ is in fact a nonlinear function of $p : L^1([0,T]; R) \to X$. Secondly (and of great importance for distributed systems) we see that (P) in fact asks us to control a (generally) infinite dimensional system with controls $p(t)$ in one-dimensional real space for each t. The first observation means that our analysis may likely resort to local theory (the inverse function theorem). The second observation is more serious since it will generally imply the map $\omega(T;\cdot,\omega_0) : L^r([0,T]; R) \to C([0,T]; X)$ is compact for $r > 1$. More precisely it was proved in [2].

Theorem 1. Let X be a Banach space with $\dim X = \infty$. If $\infty > T > 0$ and $p_n \to p$ weakly in $L^1([0,T]; R)$ then $\omega(\cdot;p_n,\omega_0) \to \omega(\cdot;p,\omega_0)$ strongly in $C([0,T]; X)$. Moreover the set of states accessible from ω_0 defined by

$$S(w_0) = \bigcup_{\substack{t>0 \\ p\in L^r_{loc}([0,\infty);R) \\ r>1}} \omega(t;p,\omega_0)$$

is contained in a countable union of compact sets of X, and in particular has dense complement.

The proof of Theorem 1 is quite technical and is given in [2]. Its importance in controllability of (1.1), (1.2) is readily seen, however. For if the set of states accessible from w_0 $S(\omega_0)$ has dense complement we shall never be able to steer to an open neighborhood in X of ω_0. In other words <u>except for an exceptional set of $h \in X$ (P) is ill-posed.</u>

3. <u>Identification of the accessible set for abstract "hyperbolic" bilinear control systems.</u>

Having noted in Theorem 1 the inability to control (1.1), (1.2) to any open neighborhood of X, we turn instead to trying to identify what states are accessible from a given $\omega_0 \in X$. Specifically we consider the abstract "hyperbolic" bilinear control system (1.3), (1.4).

As the basis $\{\phi_n\}$ of H may be regarded as a basis of $\mathcal{H}$, any $z \in \mathcal{H}$ may be expanded as a Fourier series in the basis $\{\phi_n\}$. Hence we may write

$$z(t) = \sum_{n=1}^{\infty} z_n(t)\phi_n \quad . \tag{3.1}$$

<u>We assume</u> in addition that

$$\langle B\phi_n, \phi_m \rangle_H = 0 \quad \text{for} \quad n \neq m \tag{3.2}$$

<u>and assume</u>

$$\langle B\phi_n, \phi_n \rangle_H \doteq b_n \neq 0 \quad .$$

Then substitution of (3.1) into (1.5) yields the infinite system of ordinary differential equations

$$\dot{z}_n(t) = -i\lambda_n z_n(t) - ip(t)\frac{b_n}{\lambda_n} \operatorname{Re} z_n(t), \; n = 1,2,\ldots \tag{3.3}$$

with initial conditions

$$z_n(0) = z_{0n} \quad , \quad z_{0n} \doteq \langle z(0), \phi_n \rangle \quad . \tag{3.4}$$

In itself (3.3) is not much of an improvement over the earlier formulations of our problem in that the map $p \to \{\underline{z}(T;p,\underline{z}(0))\}$ from $L^r([0,T];\mathbb{R})$ into (the natural state space for (3.3), (3.4)) $C([0,T];\ell_2)$ is still compact (now for $r > 1$). However (3.3) allows us to make the remarkable change of variables

$$\zeta_n(t) = \frac{\lambda_n}{b_n}\left[\frac{z_n(t)}{z_{0n}} \exp i\left(\lambda_n t + \frac{b_n}{2\lambda_n} P(t)\right) - 1\right] \tag{3.5}$$

where $P(t) \doteq \int_0^t p(s)ds$.

A straightforward computation shows that $\{\zeta_n(t)\}$ satisfies the equations

$$\dot{\zeta}_n(t) = i\,\frac{p(t)}{2}\,\frac{\overline{z_{0n}}}{z_{0n}}\left(\frac{b_n}{\lambda_n}\overline{\zeta}_n(t) + 1\right)\exp\left[2i\left(\lambda_n t + \frac{b_n}{2\lambda_n} P(t)\right)\right] , \tag{3.6}$$

$$\zeta_n(0) = 0,\ n = 1,2,\ldots , \tag{3.7}$$

where we assume $z_{0n} \neq 0$.

It is not hard to show (3.6), (3.7) has a solution $\{\zeta_n(t;p)\} \in C([0,T];\ \ell_2)$ which is C^1 in p as a map from $L^2([0,T];\ R)$ to ℓ_2. The amazing thing is that this map is not compact. Hence we may attempt to control (3.6) in a neighborhood in ℓ_2 of the initial state $\{0\}$. The natural way to do this is, as remarked earlier, to apply the inverse function theorem and show $D_p\{\zeta_n(T;0)\}$ (the Frechet derivative of $\{\zeta_n(T;p)\}$ with respect to p evaluated at $p = 0$) is an isomorphism from $L^2([0,T];\ R)$ to ℓ_2. Actually this won't be the case since $D_p\{\zeta_n(T;0)\}$ isn't one to one but it is onto. Fortunately the "local onto theorem" [3] will still imply the non-unique solvability of $\zeta_n(T;p) = h_n$, $n = 1,2,3,\ldots$ for some $T > 0$ and $p \in L^2([0,T];\ R)$ when $\|\{h_n\}\|_{\ell_2}$ is sufficiently small. Now knowing the accessible states of (3.6), (3.7) is an open neighborhood of $\{0\}$ in ℓ_2 we can translate back via (3.5) to find the accessible states of (3.3) from data (3.4).

For example for the rod equation with hinged ends (1.7), (1.8) $\langle B\phi_n,\phi_m\rangle_H = 0$ and assumptions (3.2) are satisfied. In fact we can prove

Theorem 2. Assume $z_{0n} \neq 0$ for the rod equation with hinged ends (1.7), (1.8). Then there exists $\varepsilon > 0$ so that if $\|\{h_n\}\|_{\ell_2} < \varepsilon$ we can solve

$$z_n\left(\frac{2}{\pi}\right) = (1 + h_n)z_{0n}, \quad n = 1,2,\ldots$$

for infinitely many $p \in L^2([0, \frac{2}{\pi}]; \mathbb{R})$ with $\int_0^{2\pi} p(t)dt = 0$.

Thus while we cannot hit all states in a small ℓ_2 neighborhood of $\{z_{0n}\}$ we can hit those of the form $\{(1 + h_n)z_{0n}\}$, $\|\{h_n\}\|_{\ell_2} < \varepsilon$. Of course using the definition of z this result could be translated into a statement (albeit messy) regarding u and u_t for (1.7), (1.8), (1.9). Furthermore using the above local result we can use a standard argument to prove

Corollary. For (1.7), (1.8) with $z_{0n} \neq 0$ $n = 1,2,\ldots$ the set of states accessible for $\{z_{0n}\}$ is dense in H.

REFERENCES

1. Ball, J. M. and M. Slemrod, Nonharmonic Fourier series and the stabilization of distributed semi-linear control systems, Communications on Pure and Applied Mathematics, 32, 555-587, (1979).

2. Ball, J. M., M. Slemrod and J. E. Marsden, Controllability for distributed bilinear systems, to appear SIAM Journal on Control and Optimization.

3. Luenberger, D. G., Optimization by Vector Space Methods, John Wiley & Sons, New York (1969).

MS/jvs

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER #2310	2. GOVT ACCESSION NO. AD-A114 625	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Distributed Bilinear Systems: Positive and Negative Results on Controllability		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) M. Slemrod		8. CONTRACT OR GRANT NUMBER(s) AFOSR-81-0172 DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 1 - Applied Analysis
11. CONTROLLING OFFICE NAME AND ADDRESS (see Item 18 below)		12. REPORT DATE December 1981
		13. NUMBER OF PAGES 7
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

and

Air Force Office of Scientific Research
Washington, DC 20332

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Distributed control system, bilinear system, hyperbolic partial differential equations, rod equation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper outlines results presented in*[2] on the problem of controlling bilinear distributed parameter systems. Specifically we give results showing that one cannot control a bilinear distributed parameter system to a full open neighborhood of an infinite dimensional state space. Nevertheless we do present a result which allows identification of an accessible set of states for a class of "hyperbolic" control systems.

* ref.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)


8